The Testmoz Invitational Mathematics Competition

June 20 – July 19, 2018

INSTRUCTIONS

1. This is a 30-question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct. The acronym NOTA means that none of the above answers are correct.
2. You will have **60** minutes to complete the test.
3. No aids are permitted other than scratch paper, graph paper, rulers, and erasers. No calculators are allowed. No problems on the test will require the use of a calculator.
4. Figures are not necessarily drawn to scale.
5. All angle measures are expressed in radians unless specified.
6. Each correct answer will be rewarded 5 points, each incorrect answer will be given 0 points, and each blank answer will be given 1 point.
7. Ties will be broken based on the hardest problem an individual answered correctly.
8. Answers will be submitted online by selecting the answer corresponding to the question number. Only submissions properly filled out on the answer form will be graded.

1. Solve for x, if $2017x + 2018 = 2018^2$.

   (A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** NOTA

2. Find the remainder when the number 2,018,000 is divided by 37.

   (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** NOTA

3. The value resulted when a number is multiplied by 5 and then increased by 2018 is the same as the value resulted when the same number is multiplied by 8 and then decreased by 7. What is the number?

   (A) 45 **(B)** 225 **(C)** 675 **(D)** 1005 **(E)** NOTA

4. Evaluate: $\dfrac{2018 \cdot 2017 - 2016 \cdot 2015}{2}$

   (A) 4029 **(B)** 4033 **(C)** 4037 **(D)** 4041 **(E)** NOTA

5. If $x \otimes y = x^4 - 20x^3 + y^2 + 150x^2 - 12y - 500x + 655$, then what is $11 \otimes 33$?

   (A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** NOTA

6. Ana rolls a fair six-sided die twice, and takes the sum of both outcomes. What is the probability she gets a sum between 6 and 8, inclusive?

   (A) $\dfrac{5}{12}$ **(B)** $\dfrac{4}{9}$ **(C)** $\dfrac{17}{36}$ **(D)** $\dfrac{1}{2}$ **(E)** NOTA

7. What is the greatest prime divisor of the number $2^{16} - 1$?

   (A) 17 **(B)** 37 **(C)** 257 **(D)** 13107 **(E)** NOTA

8. 2018_9 is equal to n in binary form (base 2). What is the sum of the digits of n?

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** NOTA

9. How many positive integer factors does $127 \cdot 128^{144}$ have?

   (A) 144 **(B)** 145 **(C)** 288 **(D)** 290 **(E)** NOTA

10. Layla, who is in a rush, drives 400 miles from a point A to a point B. Then, she drives 270 miles from point B to a point C. Finally, she drives 160 miles from point C back to point A. Layla drives 2 times as fast in her first trip than her third trip and she drives 1.5 times as fast in her second trip than her first trip. Given that the amount of time it takes Layla to take her first trip was 5 hours, what is Layla's average speed (in miles per hour) out of all three trips? Express your answer as a decimal to the nearest tenth.

    (A) 65.5 **(B)** 72.7 **(C)** 73.8 **(D)** 80.0 **(E)** NOTA

11. Daniel rows upstream at an average speed of 2 km/h, and he rows downstream at an average speed of 18 km/h. What is the speed of the current, in kilometers per hour?

    (A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** NOTA

12. What is n, if $n = \sqrt[4]{1^3 + 2^3 + 3^3 + \ldots + 8^3}$?

    (A) 6 **(B)** $2\sqrt{14}$ **(C)** 9 **(D)** $3\sqrt{5}$ **(E)** NOTA

13. Benny lives at a point on the coordinate plane that is parallel to the y-axis and passes through the point (4, 7). What is the equation and slope of this line?

    (A) $y = 7$; slope is undefined **(B)** $x = 7$; slope is undefined **(C)** $x = 7$; slope is 0
    (D) $y = \frac{7}{4}x$; slope is $\frac{7}{4}$ **(E)** NOTA

Use the information below to answer questions 14 through 16.

LiAngelo Ball, a basketball player, throws a ball up into the air. The graph of his throw can be expressed with the equation $y = -x^2 + 6x + 5$, where x and y are non-negative integers, and x represents time (in seconds), and y represents the height (in feet) from the floor.

14. From what height does he throw the ball from, in inches?

    (A) 60 **(B)** 30 **(C)** 15 **(D)** 5 **(E)** NOTA

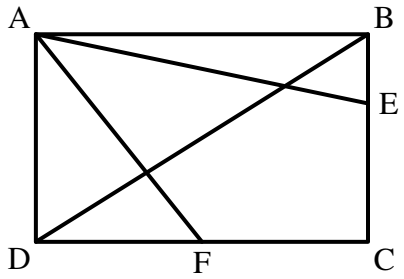
15. What is the maximum possible height, in feet, that is attained by his throw?

    (A) 3 **(B)** 5 **(C)** 14 **(D)** 26 **(E)** NOTA

16. How many seconds does it take for the ball to touch the floor for the first time?

    (A) $3 - \sqrt{14}$ **(B)** $3 + \sqrt{14}$ **(C)** 5 **(D)** 6 **(E)** NOTA

17. Rectangle $ABCD$, as shown below, is drawn such that AB has a length of 8 inches, while BC has a length of 5 inches. A segment drawn from point A intersects BC at a point E, while another segment drawn from the same point intersects CD at point F. Segments AF and AE intersect diagonal BD at G and H, respectively. Given $BE : EC = 1 : 3$ and $CF : FD = 1 : 1$, the ratio $DG : GH : HB$ is equivalent to the ratio $x : y : z$, where x, y, and z are relatively prime positive integers. What is the value of $x \cdot y \cdot z$?



    (A) 42 **(B)** 60 **(C)** 72 **(D)** 120 **(E)** NOTA

18. A regular undecagon has side length 6 units. What is the area of this polygon, in square units?

    (A) $\frac{33\cot\left(\frac{360}{11}\right)}{2}$ **(B)** $99\cot\left(\frac{360}{11}\right)$ **(C)** $\frac{33\cot\left(\frac{2\pi}{11}\right)}{2}$ **(D)** $99\cot\left(\frac{2\pi}{11}\right)$ **(E)** NOTA

19. Three real numbers are selected at random between -2 and 2, inclusive. What is the probability that the sum of these real numbers is less than or equal to 4?

    (A) $\frac{7}{8}$ **(B)** $\frac{11}{12}$ **(C)** $\frac{23}{24}$ **(D)** $\frac{47}{48}$ **(E)** NOTA

20. The shortest distance between the point $(6,-7)$ and the line defined by the equation $11x + 21y = 113$ can expressed in the form $\frac{a\sqrt{b}}{c}$, where a and c are relatively prime, and b has no perfect square factors greater than 1. What is the value of $a + b + c$?

    (A) 756 **(B)** 940 **(C)** 1216 **(D)** 2768 **(E)** NOTA

21. Evaluate $\sum_{n=1}^{m} \left(\frac{1}{2}\right)^n$ as m approaches infinity.

(A) $\frac{2}{3}$ **(B)** $\frac{3}{2}$ **(C)** 1 **(D)** ∞ **(E)** NOTA

22. Evaluate $\ln \sqrt[2]{e^{4040}} + \ln e^2 - 2$.

(A) 2014 **(B)** 2016 **(C)** 2018 **(D)** 2020 **(E)** NOTA

23. Evaluate $2\log 20 + \log 2.5 - \log 2 - \log 5$.

(A) 2 **(B)** 3 **(C)** 10 **(D)** 100 **(E)** NOTA

24. What is the magnitude of the complex number $15i - 8$?

(A) 7 **(B)** 17 **(C)** 23 **(D)** $2\sqrt{30}$ **(E)** NOTA

25. What is the product of the complex roots to the equation $x^3 = 1$?

(A) 1 **(B)** -1 **(C)** i **(D)** 0 **(E)** NOTA

26. What is the eccentricity of any parabola?

(A) 0 **(B)** $\frac{\sqrt{3}}{2}$ **(C)** 1 **(D)** $\frac{4}{3}$ **(E)** NOTA

27. Let θ be an obtuse angle such that $\sin\theta = \frac{15}{113}$. Evaluate $\sec\theta$.

(A) $\frac{112}{113}$ **(B)** $-\frac{112}{113}$ **(C)** $\frac{113}{112}$ **(D)** $-\frac{113}{112}$ **(E)** NOTA

28. Let an angle $\theta = \frac{\pi}{12}$. Evaluate $2\sin\theta\cos\theta$.

(A) $\frac{\sqrt{6}-\sqrt{2}}{2}$ **(B)** $\frac{\sqrt{6}+\sqrt{2}}{2}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{\sqrt{3}}{2}$ **(E)** NOTA

29. A particular triangle $\triangle ABC$ has $AB = 7$ units and $BC = 8$ units. Given that $\angle ABC = \arcsin(0.75)$, what is the area of triangle $\triangle ABC$, in square units?

(A) $\frac{7\sqrt{7}}{2}$ **(B)** 21 **(C)** 28 **(D)** $\frac{112}{3}$ **(E)** NOTA

30. Evaluate $\lim_{x\to 0} \frac{\sin x - x}{x^3}$.

(A) -1 **(B)** $-\frac{1}{6}$ **(C)** 1 **(D)** ∞ **(E)** NOTA