There are 30 questions of various topics below. Calculators are not recommended for practice, as the actual competition does not allow calculators. Good luck!

### **1** Number Sense and Pre-algebra Applications

- 1. What is the perimeter of a square with area 64 square units?
- 2. Evaluate  $2 \cdot (48 + 31^2)$ .
- 3. What is the sum of all the prime factors of 2018?
- 4. The value resulted when a number is multiplied by 2 and then increased by 7 is the same as the value resulted when the same number is multiplied by 5 and then decreased by 8. What is the number?
- 5. Bob has x marbles. Dale has 4x marbles. Alex has 6x marbles. Given they share 1001 marbles altogether, what is the number of marbles Dale has?
- 6. What is the value of  $5 \oplus 6$ , if  $a \oplus b = a^2 + b^2 4a + 4b + 8$ ?
- 7. Ally invests \$1000 in a bank that compounds annually by ten percent. How much money does she gain after three years?
- 8. The least common multiple of a and b is 33768, and the greatest common divisor of a and b is 36. What is the product of a and b?
- 9. Two numbers have a sum of 100 and a product of 1539. What is the sum of the squares of both numbers?
- 10. How many positive integer factors does  $2^4 \cdot 5^2 \cdot 7^3$  have?
- 11. Two angles of a triangle measure 89.9° and 65.1°. Classify this triangle as either acute, right, or obtuse.
- 12. Albert sells McNuggets strictly in packs of 5 or 12 McNuggets. What is the greatest number of McNuggets that one cannot order from Albert at a time? (One cannot order 1 McNugget, as Albert does not sell McNuggets in packs of 1. However, one can order 17 McNuggets, as they can order one pack of 5 McNuggets and one pack of 12 McNuggets.)

# 2 Counting and Probability

- 13. Trevor has two jars of identically shaped marbles. One jar contains three blue marbles and five red marbles, while the other jar contains seven blue marbles and four red marbles. He chooses a jar and picks two marbles randomly, without replacement. The probability that Trevor picks one of each type of marble can be expressed as the fraction  $\frac{m}{n}$ , where m and n are relatively prime. What is m + n?
- 14. The real numbers x and y are each randomly selected from the interval (9, 20). What is the probability that x + y > 25? Express your answer as a common fraction.

# **3** Algebra and Geometry Applications

- 15. What is the sum of the coordinates at which the lines y = 2x + 4 and 7x + y = 36 intersect?
- 16. What is the value of the y-coordinate of the intersection of the y-axis and the line passing through the points (7, 18) and (18, 95)?
- 17. What is the value of n, if  $n = \sqrt[4]{1^2 + 2^2 + 3^2 + \ldots + 8^2}$ ?
- 18. What is the area of a hexagon with side length 6 units? Express your answer in simplest radical form.
- 19. In a six by six array of points, how many squares can be formed by connecting four of the points, given each square has an integer side length?
- 20. Pablo cuts a string off a ball of yarn, and it somehow perfectly coils around a cylinder exactly once. The endpoints of the string when coiled around the cylinder are opposite each other. What is the smallest possible length of this string, given the radius of the cylinder is 3 and the height of the cylinder is  $8\pi$ ? Express your answer in terms of  $\pi$ .
- 21. TESTMOZ is a regular dodecagon with side length 7 units. The area of the circumscribed circle of TESTMOZ has an area of  $a\pi\sqrt{b} + c\pi$  square units, where b has no perfect square factors greater than 1. What is a + b + c?

# 4 Higher Algebra

22. What is  $(1+i)^{24}$ , if  $i = \sqrt{-1}$ ?

23. Evaluate 
$$\sum_{i=2}^{10} i^2$$
.

- 24. What is the eccentricity of any circle?
- 25. The distance between the point (6, -7, 8) and the plane defined by the equation 11x + 21y 25z = 113 can be expressed in the form  $\frac{a\sqrt{b}}{c}$ , for a and c are relatively prime, and b has no perfect square factors greater than 1. What is the value of a + b + c?

### 5 Basic Trigonometry

- 26. Evaluate  $\csc \alpha$ , given  $\alpha = \frac{\pi}{8}$  and that the measures are in radians.
- 27. Evaluate  $\cos^2 \alpha \sin^2 \alpha$ , given  $\alpha = \frac{\pi}{24}$  and measures are in radians.
- 28. Convert  $\frac{\pi}{2}$  degrees to radians.
- 29. Given that  $\sin \alpha = \frac{8}{17}$ , what is the value of  $\sec \alpha$ ?

### 6 Precalculus

30. Evaluate 
$$\lim_{x \to \infty} \frac{\sin x - x}{x}$$